Simulating Distributed Elements with Asymptotic Waveform Evaluation*

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Abstract

A new method for simulating distributed elements is described. The Asymptotic Waveform Evaluation (AWE) technique is used to compute a low-order approximation to the admittance matrix for a system of coupled lossy lines. The Y-matrix can serve as a macromodel for the lines, so that they can be simulated together with arbitrary (nonlinear) terminations. A major advantage of the new method is that it allows general distributed elements to be simulated in the time domain without using computationally expensive lumped models or numerical inverse transform techniques.

1. Introduction

FINER feature sizes, increasing complexity, and higher clock rates have combined to make interconnect the dominant determinant of speed in high-performance electronic systems. The ability to efficiently simulate circuits containing accurate interconnect models is essential; performance degradation due to delays, dispersion, ringing, reflection, attenuation and crosstalk must be detected before a system is built.

Models which capture the distributed nature of interconnect are thus highly desirable. Common approaches to simulating these models involve lumped networks, FFT's [2], waveform relaxation [3], or numerical inversion of the Laplace transform [9]. These methods lack the efficiency to be used in large circuit problems.

Asymptotic Waveform Evaluation (AWE) is a technique which has been developed recently for the simulation of lumped linear circuits [5]. AWE efficiently computes a reduced-order model of a very high-order linear system. It uses Padé approximation to match the $2q$ low-order time moments of the circuit response to a q^{th}-order rational function [4]. Because moment computations typically can be accomplished 100 or more times faster than a conventional transient analysis, AWE is very efficient. The result of an AWE analysis is a set of driving-point and transfer immittances for the circuit being analyzed. This information can be used as a macromodel for frequency [1] or time-domain simulations [6].

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This paper describes an extension to the basic AWE algorithm which permits the efficient analysis of circuits containing distributed elements. The distributed elements considered here are systems of coupled lossy transmission lines. First it is shown how to compute the moments of the admittance matrix for the system of lines. Then it is indicated how these moments can be used for macromodelling purposes. An example is given to demonstrate the effectiveness of the technique.

The present technique makes three improvements to previously published moment computation methods [7]. First, the derivation is more intuitive and easier to understand. Second, the method presented here does not rely on computing the derivatives of eigenvalues and eigenvectors with respect to the frequency variable s. Finally, a novel frequency-shifting technique is introduced which can lead to more accurate AWE approximations.

2. Moments of Distributed Elements

Consider a system of n coupled lossy transmission lines. The lines are described by the generalized Telegrapher's equations in matrix form:

$$\frac{\partial}{\partial x}\boldsymbol{i}(x,t) + \boldsymbol{G}\boldsymbol{v}(x,t) + \boldsymbol{C}\frac{\partial}{\partial t}\boldsymbol{v}(x,t) = 0 \qquad (1)$$

$$\frac{\partial}{\partial x}\boldsymbol{v}(x,t) + \boldsymbol{R}\boldsymbol{i}(x,t) + \boldsymbol{L}\frac{\partial}{\partial t}\boldsymbol{i}(x,t) = 0 \qquad (2)$$

Here, $\boldsymbol{i}(x,t)$ and $\boldsymbol{v}(x,t)$ represent the vectors of line currents and voltages at a point x at time t. $\boldsymbol{R}$, $\boldsymbol{G}$, $\boldsymbol{L}$, and $\boldsymbol{C}$ are the per-unit-length resistance, conductance, inductance, and capacitance matrices, respectively. Taking the Laplace transform of these equations yields

$$\frac{d}{dx}\boldsymbol{V}(x,s) + (\boldsymbol{R}+s\boldsymbol{L})\boldsymbol{I}(x,s) = 0 \qquad (3)$$

$$\frac{d}{dx}\boldsymbol{I}(x,s) + (\boldsymbol{G}+s\boldsymbol{C})\boldsymbol{V}(x,s) = 0 \qquad (4)$$

Combining (3) and (4) results in

$$\frac{d^2}{dx^2}\boldsymbol{V}(x,s) = (\boldsymbol{R}+s\boldsymbol{L})(\boldsymbol{G}+s\boldsymbol{L})\boldsymbol{V}(x,s) \qquad (5)$$

Now let $V(x, s)$ be expanded in a Taylor series about $s = 0$:

$$V(x,s) = V_0(x) + sV_1(x) + s^2V_2(x) + \ldots \quad (6)$$

Inserting (6) into (5) yields

$$\frac{d^2V_0}{dx^2} + s\frac{d^2V_1}{dx^2} + s^2\frac{d^2V_2}{dx^2} + \ldots =$$

$$(\boldsymbol{R} + s\boldsymbol{L})\,(\boldsymbol{G} + s\boldsymbol{C})\,[V_0(x) + sV_1(x) + s^2V_2(x) + \ldots] \quad (7)$$

Matching corresponding powers of s, the following set of DC differential equations is obtained:

$$\frac{d^2V_0}{dx^2} - \boldsymbol{RG}V_0(x) = 0 \quad (8)$$

$$\frac{d^2V_1}{dx^2} - \boldsymbol{RG}V_1(x) = (\boldsymbol{RC} + \boldsymbol{LG})\,V_0(x) \quad (9)$$

and for $k > 1$,

$$\frac{d^2V_k}{dx^2} - \boldsymbol{RG}V_k(x) = (\boldsymbol{RC} + \boldsymbol{LG})\,V_{k-1}(x) + \boldsymbol{LC}V_{k-2}(x) \quad (10)$$

In order to generate the moments of $V(x, s)$, the same system of ordinary differential equations must be solved repeatedly; only the forcing functions on the right-hand side change each time. First, $V_0(x)$ is determined from (8). Then (9) is used to find $V_1(x)$, and so forth, until $2q$ moments have been found.

It is possible to decouple these equations and solve them exactly if the matrix $\boldsymbol{RG}$ can be diagonalized. This can be accomplished by making a frequency-*independent* similarity transformation. The voltages $V_k(x)$ are transformed to a set of voltages $\bar{V}_k(x)$, defined by

$$V_k(x) = \boldsymbol{T}\bar{V}_k(x) \quad (11)$$

The nonsingular, constant matrix $\boldsymbol{T}$ must be chosen so that the matrix

$$\Lambda^2 = \boldsymbol{T}^{-1}(\boldsymbol{RG})\,\boldsymbol{T} \quad (12)$$

is diagonal: $\Lambda^2 = \text{diag}\,(\lambda_1^2, \lambda_2^2, \ldots \lambda_n^2)$. (The task of finding $\boldsymbol{T}$ and Λ^2 corresponds to the problem of determining the eigenvalues and eigenvectors of the constant matrix $\boldsymbol{RG}$.)

If (11) is substituted into (8)-(10), which are then premultiplied by $\boldsymbol{T}^{-1}$, one obtains

$$\frac{d^2\bar{V}_k}{dx^2} - \Lambda^2\bar{V}_k(x) = \boldsymbol{A}\bar{V}_{k-1}(x) + \boldsymbol{B}\bar{V}_{k-2}(x) \quad (13)$$

where

$$\boldsymbol{A} = \boldsymbol{T}^{-1}(\boldsymbol{RC} + \boldsymbol{LG})\,\boldsymbol{T} = [\alpha_{ij}] \quad \text{for } k > 0 \text{ and} \quad (14)$$

$$\boldsymbol{B} = \boldsymbol{T}^{-1}(\boldsymbol{LC})\,\boldsymbol{T} = [\beta_{ij}] \quad \text{for } k > 1. \quad (15)$$

It should be noted that, in general, the frequency-independent similarity transformation (12) will not also diagonalize the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. This is not a problem, however, because these matrices affect only the right-hand side of the equations.

After diagonalizing, one is left with a set of n decoupled scalar differential equations. For the i^{th} mode and k^{th} moment, $\bar{V}_k^{(i)}(x)$:

$$\frac{d^2\bar{V}_k^{(i)}}{dx^2} - \lambda_i^2\bar{V}_k^{(i)}(x) = \sum_{j=1}^{N}\alpha_{ij}\bar{V}_{k-1}^{(j)}(x) + \sum_{j=1}^{N}\beta_{ij}\bar{V}_{k-2}^{(j)}(x) \quad (16)$$

The right-hand side of (16) contains contributions from lower-order moments of modes other than the i^{th}. This does not invalidate the decoupling, however, because the lower-order moments will already have been computed. The homogeneous solution to this equation is simply determined from the boundary conditions at $x = 0$ and $x = l$:

$$\bar{V}_{k,h}^{(i)}(x) = \bar{V}_k^{(i)}(0)\frac{\sinh\lambda_i(l-x)}{\sinh\lambda_i l} + \bar{V}_k^{(i)}(l)\frac{\sinh\lambda_i x}{\sinh\lambda_i l} \quad (17)$$

where l is the length of the lines. The particular solution is slightly more difficult to obtain. Since the forcing functions are also solutions of the homogeneous equation, the particular solution will involve not only sinh() and cosh(), but also powers of x multiplying these functions:

$$\bar{V}_{k,p}^{(i)}(x) = \sum_{m=0}^{k}\sum_{j=1}^{n} M_{ij}^{km}x^m\cosh(\lambda_j x) + \sum_{m=0}^{k}\sum_{j=1}^{n} N_{ij}^{km}x^m\sinh(\lambda_j x) \quad (18)$$

By substituting (18) into (16), and matching like powers of x, recursion relationships between the coefficients M_{ij}^{km} and N_{ij}^{km} can be derived. Thus it is possible to derive exact analytical formulae for the moments of the line voltage, without relying on any form of lumped (discrete) approximation.

Once the moments $\bar{V}_k(x)$ have been determined, the moments $V_k(x)$ can be found by premultiplying by $\boldsymbol{T}$. Then, in order to get the moments of the admittance matrix, the moments of $\boldsymbol{I}(x, s)$ must be determined. It is assumed that $\boldsymbol{I}(x, s)$ has a Taylor series about $s = 0$:

$$\boldsymbol{I}(x,s) = \boldsymbol{I}_0(x) + s\boldsymbol{I}_1(x) + s^2\boldsymbol{I}_2(x) + \ldots \quad (19)$$

Substituting this into (3) and matching like powers of s yields

$$\frac{d\boldsymbol{V}_0(x)}{dx} + \boldsymbol{R}\boldsymbol{I}_0(x) = 0 \quad (20)$$

and for $k > 1$,

$$\frac{d\boldsymbol{V}_k(x)}{dx} + \boldsymbol{R}\boldsymbol{I}_k(x) + \boldsymbol{L}\boldsymbol{I}_{k-1}(x) = 0 \quad (21)$$

Since the moments of $V(x, s)$ as well as the lower-order moments of $I(x, s)$ are known, (21) can be solved for the next moment $I_k(x)$. In most cases, only the "port" currents at the points $x = 0$ and $x = l$ are of interest; so (20) and (21) need only be solved at these points. Of course, since the $V_k(x)$ are known analytically, it is a simple matter to compute their derivatives.

The procedure described above allows one to compute the moments of the multiport admittance matrix $Y(s)$ for a coupled system of lossy lines:

$$Y(s) = Y_0 + sY_1 + s^2Y_2 + \ldots \tag{22}$$

Once the moments have been found, they can be used as stencils within a global Y-matrix [1]. This allows large linear circuits containing both lumped and distributed elements to be simulated efficiently in the frequency domain. Alternatively, if nonlinear terminations are present, time-domain simulation may be preferred. Padé approximation may be used to convert the $2q$ low-order moments for each matrix entry (i,j) into a q-pole rational function [4]. This can be written in partial fraction form as

$$\sum_{m=1}^{q} \frac{k_m}{s - p_m} = [Y_0]_{ij} + s[Y_1]_{ij} + s^2[Y_2]_{ij} + \ldots \tag{23}$$

where the p_m are poles of the approximation, and the k_m are the corresponding residues. It should be noted that different orders of approximation q can be used for each entry of the matrix. Also, the poles are not restricted to be common among the entries. This freedom permits each entry to be approximated more accurately. The (i,j) entry of the approximate impulse response matrix for the lines can then be determined easily by taking the symbolic inverse transform of (23):

$$y_{ij}(t) = \sum_{m=1}^{q} k_m \exp(p_m t) \tag{24}$$

The impulse responses can be used to derive a matrix stencil for the system of lines. This stencil can be employed in a time-domain simulator based on conventional Newton-Raphson and numerical integration techniques. Only a constant number of operations are required at each time point to perform the stencilling. This capability has been implemented in the AWESpice program [6]. An analogous technique has been employed outside of the SPICE context in [8].

3. Frequency Shifting

In some cases, using the expansion point $s = 0$ for the Taylor series results in a poor Padé approximation. It is often possible to improve the approximation by choosing a different expansion point along the positive part of the real axis. This change of origin helps to reduce problems caused by poles with vastly different magnitudes (stiff systems.) To perform the expansion about a point $s = h$, $h > 0$, rewrite (3) and (4) as

$$\frac{d}{dx}V(x, s) + [R + hL + (s - h)L]I(x, s) = 0 \tag{25}$$

$$\frac{d}{dx}I(x, s) + [G + hC + (s - h)C]V(x, s) = 0 \tag{26}$$

It is now possible to define frequency-shifted R and G matrices as

$$\begin{aligned} R' &= R + hL \\ G' &= G + hC \end{aligned} \tag{27}$$

$V(x, s)$ and $I(x, s)$ can be expanded in terms of the variable $s - h$:

$$\begin{aligned} V(x, s) &= \hat{V}_0(x) + (s - h)\hat{V}_1(x) + (s - h)^2\hat{V}_2(x) + \ldots \\ I(x, s) &= \hat{I}_0(x) + (s - h)\hat{I}_1(x) + (s - h)^2\hat{I}_2(x) + \ldots \end{aligned} \tag{28}$$

By substituting (27) and (28) into (25) and (26), it is possible to derive recursion equations for the frequency-shifted moments. These equations need not be presented here, because they are identical in form to the relations derived in Section 2. Once the frequency-shifted moments of the admittance matrix have been determined, Padé approximation may again be applied. The resulting approximate, shifted poles may easily be converted back to the original origin through addition of the real constant h.

Empirical evidence shows that frequency shifting can substantially improve the quality of the Padé approximation. In addition, frequency-shifting also permits handling the special cases of lines with zero resistance or conductance matrices. The shifted matrices R' and G' will always be nonzero, and thus the method described in Section 2 applies without modification.

4. Example

To demonstrate their effectiveness, the techniques presented in this paper were applied to a circuit containing two systems of coupled lossy lines, lumped elements and nonlinear 54F240 line drivers. The circuit is shown below in Figure 1. Note that one distributed element consists of four coupled lossy lines, and the other of two. Each 54F240 driver model contains 7 transistors and 14 diodes.

The moment computations for the distributed elements required a total of 0.6 CPU-sec. The transient analysis of the circuit using the AWESpice program [6] consumed 16.6 more CPU-sec. (All CPU times are for a DECstation 3100). The same simulation—using a *four-segment* approximation to each of the systems of lines—required 23.4 CPU-sec. The resulting waveforms at the source and load points are shown in Figure 2 below. An example of a crosstalk waveform from the same circuit is shown in Figure 3.

5. Conclusions

AWE provides an efficient mechanism for developing compact macromodels of distributed circuit elements. A new method for accomplishing the moment calculations has been presented which relies on simple recursive solution of ordinary differential equations. No lumped approximations are required. A novel frequency-shifting method has also been described. This

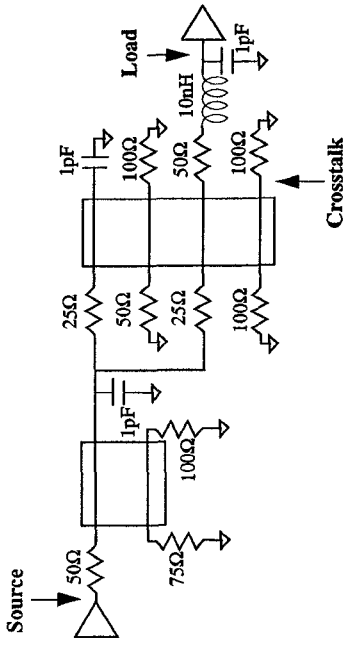

Figure 1. Example Circuit

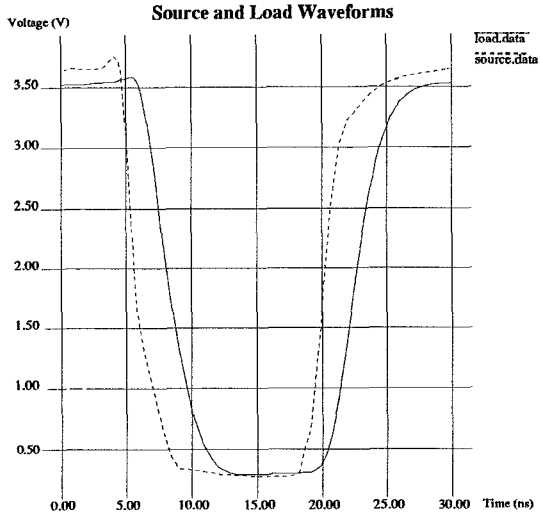

Figure 2. Source and Load Waveforms

is helpful in obtaining improved Padé approximations.

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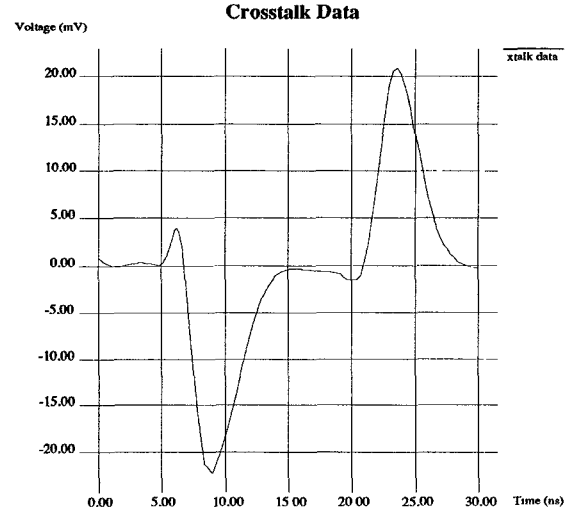

Figure 3. Crosstalk Waveform

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